The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: August 3, 2016 Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and homework solution sets.
- **Power off all cell phones**
- You may use any standalone calculator or other computing system, i.e. one that is not connected to a network.
- Please do not wear hats or headphones during the exam.
- All work should be performed on the exam itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

Problem	Point Value	Your score	Topic
1	21		Continuous-Time Signals
2	21		Continuous-Time Systems
3	21		Discrete-Time Signals
4	21		Discrete-Time Systems
5	16		Transform Connections
Total	100		

Problem 2.1 Continuous-Time Signals. *21 points*.

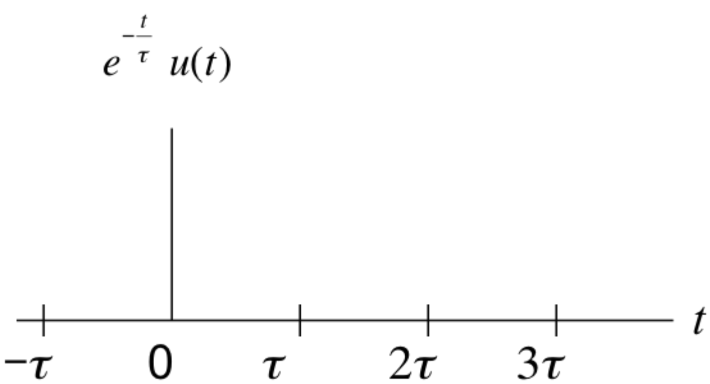
The unilateral Laplace transform transforms a continuous-time function $x(t)$ into a function $X(s)$ of a complex-variable s as follows:

$$X(s) = \int_{0^-}^{\infty} x(t) e^{-st}\, dt$$

(a) Using the Laplace transform definition above, find the Laplace transform of the continuous-time impulse $\delta(t)$. Please include the region of convergence. *6 points*.

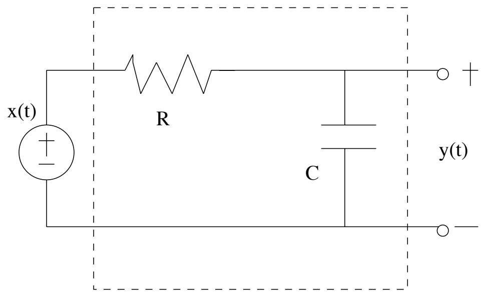
(b) Using the Laplace transform definition above, find the Laplace transform of a causal exponential signal $\exp(-t / \tau)\, u(t)$ where τ is the time constant in units of seconds where $\tau > 0$ and $u(t)$ is the unit step function. Please include the region of convergence. *9 points*.

(c) Below, plot by hand the causal exponential signal $\exp(-t / \tau)\, u(t)$ where $\tau > 0$ vs. time t. *6 points*.



Problem 2.2 Continuous-Time Systems. *21 points*.

Consider the following analog continuous-time circuit with input $x(t)$ and output $y(t)$:



Analyze the circuit for $t > 0^-$ given that the initial voltage across the capacitor is 0 Volts.

(a) Draw the circuit in the Laplace domain. *6 points*.

(b) Give the transfer function $H(s)$ in the Laplace domain. *6 points.*

(c) Based on your answer in (b), find the impulse response $h(t)$. *6 points*.

(d) Give a formula for the system time constant in terms of R and C. *3 points*.

Problem 2.3 Discrete-Time Signals. *21 points*.

The unilateral z-transform transforms a discrete-time function $x[n]$ into a function $X[z]$ of a complex-variable z as follows:

$$X[z] = \sum_{n=0}^{\infty} x[n]\, z^{-n}$$

(a) Using the z-transform definition above, find the z-transform of the discrete-time impulse $\delta[n]$. Please include the region of convergence. *6 points*.

(b) Using the z-transform definition above, find the z-transform of a causal exponential signal $a^n\, u[n]$ where a is a complex-valued constant. Please include the region of convergence. *6 points*.

(c) Using z-transforms, compute the convolution $v[n] = a^n\ u[n]\ *\ b^n\ u[n]$ where a and b are constants so that $a \neq b$. Please track the region of convergence in your calculations. *9 points*.

Problem 2.4 Discrete-Time Systems. *21 points*.

Consider a causal discrete-time linear time-invariant system with input $x[n]$ and output $y[n]$ being governed by the following difference equation:

$$y[n] = (2 \cos \omega_0)\, y[n-1] - y[n-2] + x[n] - (\cos \omega_0)\, x[n-1]$$

The impulse response is a *causal sinusoid* with constant discrete-time frequency ω_0 in units of rad/sample.

(a) Please state all initial conditions. Please give values for the initial conditions to satisfy the stated system properties. *6 points*.

(b) Find the equation for the transfer function $H[z]$ in the z-domain, including the region of convergence. *9 points*.

(c) Find the inverse z-transform of the transfer function in part (b) to find the formula for the impulse response $h[n]$ of the system. *6 points*.

Problem 2.5 Transform Connections. *16 points*.

The unilateral Laplace transform transforms a continuous-time function $x(t)$ into a function $X(s)$ of a complex-variable s as follows:

$$X(s) = \int_{0^-}^{\infty} x(t) e^{-st}\, dt$$

The unilateral z-transform transforms a discrete-time function $x[n]$ into a function $X[z]$ of a complex-variable z as follows:

$$X[z] = \sum_{n=0}^{\infty} x[n]\, z^{-n}$$

Consider the following mapping from the s-plane to the z-plane

$$z = e^{s\,T}$$

where T is a positive constant ($T > 0$) with units in seconds.

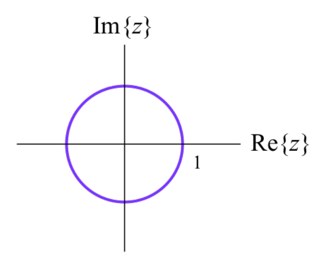
(a) For the above mapping $z = \exp(s\,T)$, write the z variable in polar form $r \exp(j\,\omega)$ and the s variable in Cartesian form $\sigma + j\,\Omega$. The variable Ω is a frequency in units of rad/s.

i. Give the formula for r in terms of σ, Ω and T. *3 points*.

ii. Give the formula of ω in terms of σ, Ω and T. *3 points*.

(b) To where in the z-plane would the following values of s-plane map?

i. $\text{Re}\{s\} = 0$. Imaginary axis of the s-plane, i.e. $s = j\,\Omega$. *4 points*.



z-plane with
circle of radius 1

ii. $\text{Re}\{s\} < 0$. Left-hand side of the s-plane. *3 points*

iii. $\text{Re}\{s\} > 0$. Right-hand side of the s-plane. *3 points*.